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Zener tunnelling and Bloch oscillations in random external fields

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Abstract. Within a two-band tight-binding model, we investigate the dynamics of electrons on which a stochastic external electric field is imposed. We find that the external noise may destroy the Zener resonance, the Bloch oscillation, and the miniband localization. The destruction of the Zener resonance due to the noise is much easier than that of the Bloch oscillations.

1. Introduction

In ordinary bulk materials, subjected to accessible electric fields, the Bloch oscillations (BOs) [1] cannot be observed, because scattering typically disrupts the coherent motion of electrons on a timescale which is much shorter than the Bloch period under practical conditions. Thus, for a long time the prediction of BOs could not be verified. However, the emergence of semiconductor superlattices brings about a dramatic change of the conditions, because a semiconductor superlattice has a larger lattice constant (d) and correspondingly small Bloch period (T_B). Recently, experimental advances have been made in the systems of semiconductor superlattices; these include the observation [2] of Bloch oscillations and studies of photon-assisted transport [3–5]. These experimental results have not only given verifications of the classic predictions of Bloch and Zener [6], but also opened up a new interesting field for theoretical and experimental work.

On the theoretical side, a great deal of interest [7–22] has been shown in the influence of external electric fields on semiconductor superlattices. Many new phenomena have been predicted/explained theoretically, such as negative differential conductivity [14, 15], dynamic localization [16–20], band collapse [7, 21, 22], band suppression [10], and fractional Wannier–Stark ladders [8]. The simplest model to use when investigating semiconductor superlattices involves just a single miniband; this can explain/predict a lot of new phenomena, but misses all of the interesting interband effects. Thus, a two-band tight-binding model [23] is used for theoretically investigating semiconductor superlattices. This model contains the essential physics of interband transitions, and can be compared in practice with realistic situations [24–26] where only a single pair of bands is important.

Rotvig, Jauho, and Smith (RJS) [27, 28] have studied the coherent transport of one-dimensional semiconductor superlattices under the action of an electric field within this model. They found that the coherent oscillation between the minibands can occur at special values of the applied static electric field, where there are avoided crossings of the two interpenetrating Wannier–Stark ladders (WSL) arising from different bands. These are the so-called Zener resonances. For electric field values between the Zener resonances there are stable plateaus, where the relative population of the two bands is only weakly dependent on time. In each plateau there are small-amplitude oscillations. The lifetime of a plateau is equal to the Bloch period. On the other hand, they found that the Zener resonances are sensitive to the superlattice parameters and the external electric field, and a system of Zener resonances can be changed into a system of miniband localization by adjusting the value of the external field.

Using the same model, Zhao *et al* [29] investigated the dynamics of electrons with Markovian dephasing under the influence of static fields and the effects of scattering from lattice imperfections, using a stochastic Liouville equation for the density matrix. The results showed that the dephasing ultimately takes electrons that are initially located in one miniband to equal population of the two minibands, instead of them undergoing persistent Rabi flop, as they do in the absence of scattering.

In practice, since the external field may have a fluctuation component, the effect of external noise should also be considered. In this work, using the same model, we study the dynamics of electrons when the external field has a fluctuating component. As a versatile choice for the noise, we will use an Ornstein–Uhlenbeck (OU) process [30]. This will permit an investigation of the role of the strength of the noise and the size of the correlation time of the noise on the time evolution of the population between two minibands. By means of numerical calculations, we can obtain the time evolution of $\rho_-(k,t)$, which is the difference in electron population of the two minibands in the quasimomentum space. We find that both BOs and Zener resonances at avoided crossings may be destroyed by the external noise, and that the destruction of the Zener resonances occurs much more readily than that of BOs. Therefore for a two-band system with external noise perturbation, the observation of BOs should be easier than that of the Zener resonances.

2. Model and method

We use the standard tight-binding model of a two-band system in an electronic field $E(t)$. The model Hamiltonian [23] can be written as

$$H(t) = \sum_n [(\Delta_a + neE(t)d)a_n^+ a_n + (\Delta_b + neE(t)d)b_n^+ b_n - (W_a/4)(a_{n+1}^+ a_n + \text{h.c.}) + (W_b/4)(b_{n+1}^+ b_n + \text{h.c.}) + eE(t)R(a_n^+ b_n + b_n^+ a_n)]. \tag{1}$$

Here the integer n labels the lattice sites and the operators a and b refer to electrons in the lower and upper minibands, respectively. The first two terms describe the site energies of the Wannier states in the presence of the electric field, and $W_{a,b}$ are the widths of the isolated ($E = 0$) minibands induced by the nearest-neighbour hopping:

$$\epsilon^{a,b}(k) = \Delta_{a,b} \mp (W_{a,b}) \cos(kd)$$

where d is the lattice constant. The last term is the on-site electric dipole coupling between minibands; eR is the corresponding dipole moment. This Hamiltonian does neglect Coulomb interactions and electric dipole elements between Wannier states on different sites, but it contains the essential physics of the problem [23, 25, 27, 28].

When we apply a static electric field E_0 to the system, there will be a stochastic component $F(t)$ fluctuating around E_0 due to the circuitry thermal fluctuations. Therefore, the total electric field $E(t)$ contains two components, the stochastic part $F(t)$ and the static part E_0, i.e.,

$$E(t) = E_0 + F(t). \tag{2}$$

The magnitude of the noise $F(t)$ is usually of the order of ten per cent of E_0, which can be estimated as follows.

Consider a semiconductor superlattice of length l, average dielectric constant ϵ, and area S that is shunted by an external measuring device of resistance R_d [31], and a dc voltage U_0 ($=-lE_0$) that is imposed on the system by one constant-current source. Due to the circuitry thermal fluctuations, we have a Langevin equation for this resistively shunted semiconductor superlattice:

$$C\frac{\mathrm{d}U(t)}{\mathrm{d}t}+\frac{U(t)}{R_d}=I_0+I_n(t) \tag{3}$$

where $U(t)=U_0+U'(t)$, $U'(t)$ is the thermal fluctuation voltage across the semiconductor superlattice, $I_n(t)$ is the thermal fluctuation current, and $C=\epsilon S/l$ is effective capacitance of the semiconductor superlattice. Since $I_0=U_0/R_d$, equation (3) turns out to be

$$\frac{\mathrm{d}U'(t)}{\mathrm{d}t}+\frac{U'(t)}{CR_d}=\frac{I_n(t)}{C}. \tag{4}$$

The $I_n(t)$ can be taken as white noise, i.e.

$$\langle I_n(t)\rangle=0 \qquad \langle I_n(t_1)I_n(t_2)\rangle=2R_d^{-1}k_BT\delta(t_1-t_2)$$

where k_B is the Boltzmann constant and T is the lattice temperature. Thus, considering that the strength of the electric field fluctuations $F(t)=-U'(t)/l$, and using the method of reference [32], we can obtain the electric field correlation function

$$\langle F(t_1)F(t_2)\rangle=\frac{k_BT}{Cl^2}\mathrm{e}^{-|t_2-t_1|/R_dC} \tag{5}$$

which is equivalent to an OU process [33] whereby the noise $F(t)$ has zero average value and the correlation function

$$\langle F(t)F(s)\rangle=\Delta^2\exp(-|t-s|/\tau_c). \tag{6}$$

Here we have introduced $\tau_c=R_dC$ and

$$\Delta=\left(\frac{k_BT}{Cl^2}\right)^{1/2}=\left(\frac{k_BT}{\epsilon V_s}\right)^{1/2}$$

where V_s is the volume of the semiconductor superlattice. From this result we find that when V_s is approximately of the order of $\mu\mathrm{m}^3$, Δ is approximately of the order of 10^4 V m^{-1} at $T=300$ K. This leads to $\Delta/E_0\sim 0.1$, for a typical order of E_0. As we will see in the following, this order will result in there being significant changes in the dynamic behaviour of the Zener tunnelling and Bloch oscillations.

Now we define the density matrix in the two-band representation:

$$\rho(t)=\sum_{ijmn}\rho_{mn}^{ij}\xi_m^{i\dagger}\xi_n^j \tag{7}$$

where $i,j=1$ or 2 are band indices, and $\xi_m^{1\dagger}$ (ξ_m^1) and $\xi_m^{2\dagger}$ (ξ_m^2) designate $a_m^\dagger$ (a_m) and $b_m^\dagger$ (b_m), respectively. The density matrix $\rho(t)$ satisfies the following Liouville equation (LE) (we set $\hbar=1$ throughout this paper):

$$\mathrm{i}\frac{\partial\rho}{\partial t}=[H,\rho(t)]. \tag{8}$$

Since we are interested in the dynamics of occupation of various band states, it is convenient to work in a wave-vector basis, by Fourier transforming the density matrix. In general, since $\rho_{m,n}$ is not translationally invariant (a function only of $m-n$), we have a full set

$$\rho_{kq}^{ij}=\sum_{mn}\rho_{mn}^{ij}(t)\exp(-\mathrm{i}km+\mathrm{i}qn)$$

of Fourier components. But we will be interested in the wave-vector diagonal band occupation numbers $\rho_{kk}^{ij} \equiv \rho^{ij}(k,t)$. These evolve according to the corresponding Fourier transform of the LE:

$$\mathrm{i}\frac{\partial}{\partial t}\rho^{11}(k,t) = \mathrm{i}edE(t)\frac{\partial}{\partial k}\rho^{11}(k,t) - eE(t)R(\rho^{12}(k,t) - \rho^{21}(k,t)) \tag{9}$$

$$\mathrm{i}\frac{\partial}{\partial t}\rho^{22}(k,t) = \mathrm{i}edE(t)\frac{\partial}{\partial k}\rho^{22}(k,t) + eE(t)R(\rho^{12}(k,t) - \rho^{21}(k,t)) \tag{10}$$

$$\mathrm{i}\frac{\partial}{\partial t}\rho^{12}(k,t) = \mathrm{i}edE(t)\frac{\partial}{\partial k}\rho^{12}(k,t) - eE(t)R(\rho^{11}(k,t) - \rho^{22}(k,t)) + (\Delta_{ab} - W\cos k)\rho^{12}(k,t) \tag{11}$$

$$\mathrm{i}\frac{\partial}{\partial t}\rho^{21}(k,t) = \mathrm{i}edE(t)\frac{\partial}{\partial k}\rho^{21}(k,t) + eE(t)R(\rho^{11}(k,t) - \rho^{22}(k,t)) - (\Delta_{ab} - W\cos k)\rho^{21}(k,t) \tag{12}$$

where we have adopted the notation $\Delta_{ab} \equiv \Delta_a - \Delta_b$ and $W = (W_a + W_b)/2$. By introducing [29]

$$\begin{aligned}
\rho_+(k,t) &= \rho^{11}(k,t) + \rho^{22}(k,t)\\
\rho_-(k,t) &= \rho^{11}(k,t) - \rho^{22}(k,t)\\
\rho_{+-}(k,t) &= \rho^{12}(k,t) + \rho^{21}(k,t)\\
\rho_{-+}(k,t) &= \mathrm{i}[\rho^{21}(k,t) - \rho^{12}(k,t)]
\end{aligned}$$

we obtain

$$\frac{\partial}{\partial t}\rho_+(k,t) - edE(t)\frac{\partial}{\partial k}\rho_+(k,t) = 0 \tag{13}$$

$$\frac{\partial}{\partial t}\rho_-(k,t) - edE(t)\frac{\partial}{\partial k}\rho_-(k,t) = -2eE(t)R\rho_{-+}(k,t) \tag{14}$$

$$\frac{\partial}{\partial t}\rho_{+-}(k,t) - edE(t)\frac{\partial}{\partial k}\rho_{+-}(k,t) = (\Delta_{ab} - W\cos k)\rho_{-+}(k,t) \tag{15}$$

$$\frac{\partial}{\partial t}\rho_{-+}(k,t) - edE(t)\frac{\partial}{\partial k}\rho_{-+}(k,t) = -(\Delta_{ab} - W\cos k)\rho_{+-}(k,t) + 2eE(t)R\rho_-(k,t). \tag{16}$$

The equation for $\rho_+(k,t)$ describes particle conservation. It is decoupled from the others, and we will ignore it in the following discussion. The equations for $\rho_-(k,t)$, $\rho_{+-}(k,t)$, and $\rho_{-+}(k,t)$ can be reduced to the following ordinary differential equations in an accelerated basis [34], $k(t) = k - A(t)$, where

$$A(t) \equiv \int_0^t edE(t)\,\mathrm{d}t.$$

For the case of the external electric field equation (2), and terms of dimensionless variables $t' = edE_0t \equiv \omega_B t$ (we do not indicate the prime, for notational convenience), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}X(k,t) = -2\frac{R}{d}[1 + v(t)]z(k,t) \tag{17}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}Y(k,t) = \left\{\frac{\Delta_{ab}}{\omega_B} - \frac{W}{\omega_B}\cos\left[k - t - \int_0^t v(t)\,\mathrm{d}t\right]\right\}Z(k,t) \tag{18}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}Z(k,t) = -\left\{\frac{\Delta_{ab}}{\omega_B} - \frac{W}{\omega_B}\cos\left[k - t - \int_0^t v(t)\,\mathrm{d}t\right]\right\}Y(k,t) + 2\frac{R}{d}[1 + v(t)]X(k,t). \tag{19}$$

Here,

$$X(k,t) = \rho_{-}(k - A(t), t)$$
$$Y(k,t) = \rho_{+-}(k - A(t), t)$$
$$Z(k,t) = \rho_{-+}(k - A(t), t).$$

$\omega_B = edE_0$, and $v(t) = F(t)/E_0$ is the OU process with correlation

$$\langle v(t)v(s)\rangle = (D/\omega_B)^2 \mathrm{e}^{-|t-s|/\omega_B\tau_c} \tag{20}$$

where $D = ed\Delta$.

These stochastic equations of motion, equations (17)–(19) and (20), can be solved numerically by generating trajectories for the different realizations of the noise. In the absence of any scheme for solving these equations analytically, we turn in the following section to numerical solutions. The procedure for integrating the stochastic equations is as follows. A Runge–Kutta integrator is used to advance the solution of equations (17)–(19) for each time step, and the accompanying initial conditions are $X(k,0) = 1$, $Y(k,0) = 0$, and $Z(k,0) = 0$. A stochastic term is added at each step with its statistical properties described by an OU process. The OU process is generated by solving a Langevin equation with a delta-correlated noise term. This will ensure that the correlation function of $F(t)$ or $v(t)$ has the desired statistical property given by equation (6) or equation (20). The procedure is carried out for a sufficiently large number of trajectories to yield the desired average behaviour. The details of the method can be seen in the appendix of reference [33].

3. Results and discussion

Before displaying our calculation results we give a brief review of RJS's main results from reference [27].

(i) The Zener resonances. For certain field values $E_0 \approx \Delta_{ab}/edn$, where n is an integer, the two Stark ladders are very close to each other, forming avoided crossings, and in the corresponding neighbourhoods a strongly enhanced band-to-band transfer takes place. The time dependence of the relative population of the two bands (for a fixed k) oscillates between +1 and −1, and a set of stable plateaus form. The Zener resonances can be indexed with n, and the number of oscillations on a given plateau is equal to n.

(ii) Miniband localizations. When the field values are adjusted far away from the avoided crossings, the Zener resonances will be quenched.

The above picture is valid when the external field is a steady static one, while in practice it may have a fluctuating component. Hence, a natural question arises: what will happen when the noise field is taken into account? To answer this question, we will investigate the influence of the noise on $X(k,t)$ in three cases in this section, in which the part of the static electric field is chosen to be the value corresponding to the eighth Zener resonance, the second Zener resonance, and the miniband localization [27], respectively. The time dependence of $X(k,t)$ is obtained from equations (17)–(19) and equation (20) by numerical integration.

Figure 1 shows the time dependencies of $X(k = 0, t)$ in the absence of noise and in the case of weak noise with $D/\omega_B = 0.05$ and $\omega_B\tau_c = 0.1$. There, the superlattice parameters $W = (W_a + W_b)/2 = 18$ meV, $\Delta_{ab} = \Delta_a - \Delta_b = 20$ meV, $R/d = 0.9$, and the static part of the electric field $\omega_B = eE_0d = 2.32$ meV are chosen to be at an avoided crossing of the interpenetrating WSL. The static part of the electric field is that corresponding to the eighth Zener resonance. In the absence of noise, $X(k = 0, t)$ oscillates between +1 and −1 with a period approximately determined by a value given in reference [26]. A distinctive set of

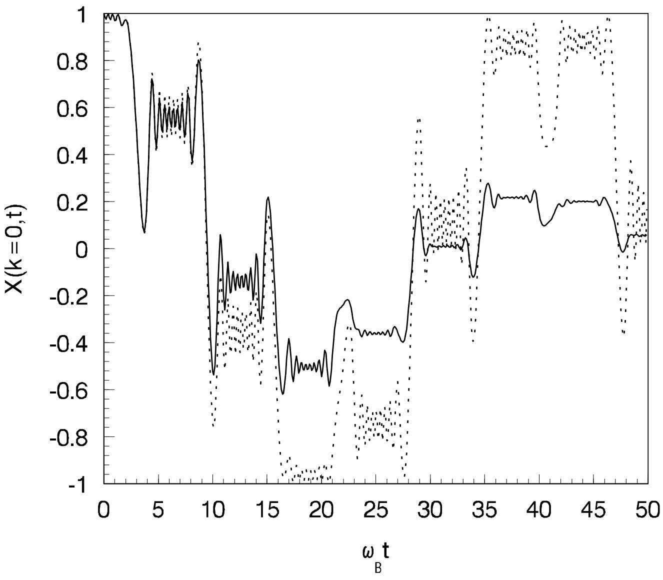

Figure 1. $X(k = 0, t)$ as a function of dimensionless time $\omega_B t$ with and without external noise. $W = (W_a + W_b)/2 = 18$ meV, $\Delta_{ab} = \Delta_a - \Delta_b = 20$ meV, $R/d = 0.9$, $\omega_B = 2.32$ meV, $D/\omega_B = 0.05$, and $\omega_B \tau_c = 0.1$. The solid line corresponds to the case of weak noise, and the dotted line to the case without external noise.

stable plateaus have developed, and the transitions between the plateaus occur at the instants $t = \frac{1}{2}T_B, \frac{3}{2}T_B, \frac{5}{2}T_B, \ldots (T_B = 2\pi/\omega_B)$. Thus, the lifetime of a plateau is (approximately) equal to the Bloch period, and transitions occur every time a k-point reaches the Brillouin zone edge, where the energy separation between the minibands is at a minimum. The number of small oscillations on a given plateau is equal to eight, meaning that in a period of Bloch oscillations there are eight periods of the small-amplitude oscillations. In the case of weak noise, $X(k = 0, t)$ still oscillates between negative and positive values, but the oscillation amplitude decreases strikingly with increasing time. The peaks of the transitions between the plateaus almost occur at the same time position as in the absence of noise. With the time increasing, the small oscillations on the plateaus become very weak, but the peaks of the transitions can still be identified. From figure 1 one can see that the transition of electrons from one miniband to the other miniband is mainly attributable to the Zener tunnelling at the zone edges, and that electrons can last for a few Zener tunnelling periods before they are scattered.

When the strength of the noise increases to $D/\omega_B = 0.2$, we find that the Zener resonance is suppressed heavily as shown in figure 2, where $X(k = 0, t)$ remains positive throughout the whole driving process. The plateaus can still be identified in earlier peaks of the envelopes, and the transitions between the plateaus can still be seen obviously about at $t = \frac{1}{2}T_B, \frac{3}{2}T_B, \frac{5}{2}T_B, \ldots$, but the small oscillations on the plateaus almost disappear. $X(k = 0, t)$ on every plateau is only weakly dependent on time. From figure 2 one can see that electrons can last for several periods of the BOs, but the electrons cannot last for a period of Zener resonances before they are scattered by the noise. For the case of strong noise ($D/\omega_B = 1.0$, $\omega_B \tau_c = 1.0$), we find that not only the Zener oscillation but also the plateaus are destroyed almost completely, as shown in figure 3. $X(k = 0, t)$ decays almost exponentially to zero with increasing time, implying that the electron density will be distributed equally between the two minibands. In that case the electron cannot last for a period of BOs before it is scattered by the noise.

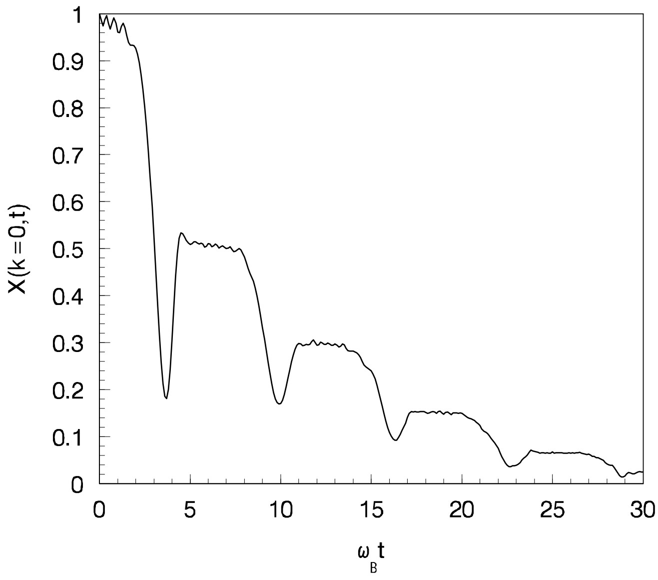

Figure 2. $X(k=0,t)$ with external noise as in figure 1, but for $D/\omega_B = 0.2$.

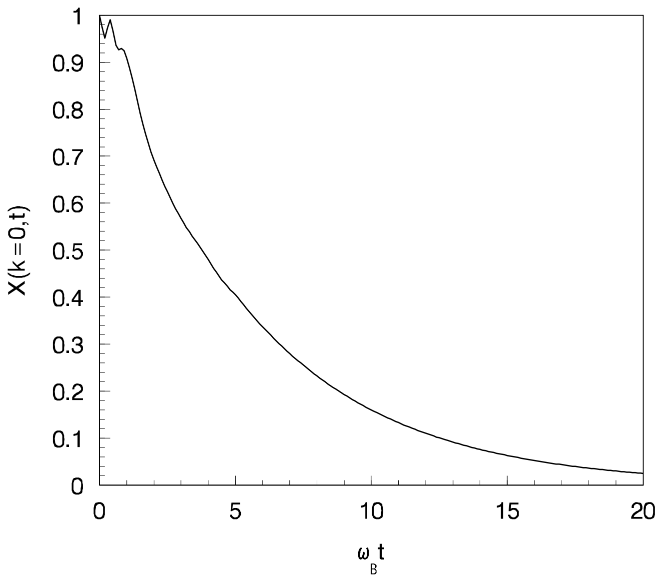

Figure 3. $X(k=0,t)$ with external noise as in figure 1, but for $D/\omega_B = 1.0$ and $\omega_B\tau_c = 1.0$.

Figure 4 shows the time dependencies of $X(k=0,t)$ in the absence of noise and for the case of weak noise with $D/\omega_B = 0.1$, $\omega_B\tau_c = 0.1$, where the superlattice parameters $W = 8.6$ meV, $\Delta_{ab} = 20$ meV, $R/d = 0.18$, and the static part of the electric field $\omega_B = 10.2$ meV are chosen to be at an avoided crossing of the interpenetrating WSL. The static part of the electric field in this case is that corresponding to the second Zener resonance. In the absence of noise, $X(k=0,t)$ oscillates between +1 and -1; one can distinguish two periods

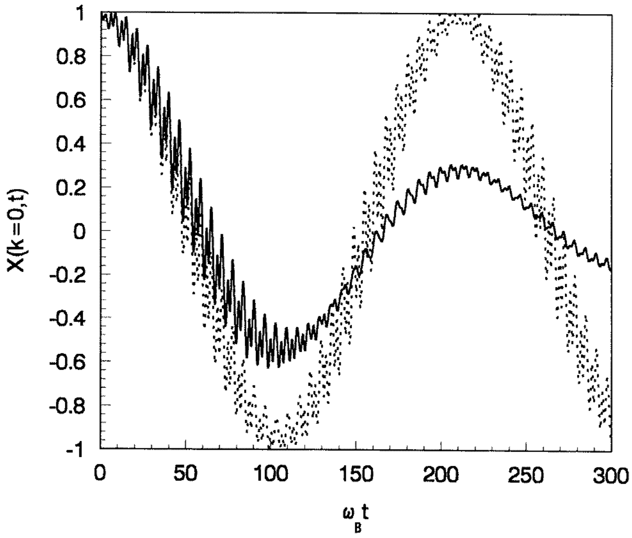

Figure 4. $X(k = 0, t)$ as a function of dimensionless time $\omega_B t$ with and without external noise. $W = 8.6$ meV, $\Delta_{ab} = 20$ meV, $R/d = 0.18$, $\omega_B = 10.2$ meV, $D/\omega_B = 0.1$, and $\omega_B \tau_c = 0.1$. The solid line corresponds to the case of weak noise, and the dotted line to the case without external noise.

of oscillations in any of the plateaus (even though the plateaus are not very clearly resolved for this particular set of parameters). In the case of weak noise, $X(k = 0, t)$ still oscillates between negative and positive values, and the oscillation amplitude decreases strikingly with increasing time. The oscillations on the plateaus can be identified in several earlier plateaus, and their oscillation amplitude decreases with increasing time. When the strength of the noise increases to $D/\omega_B = 0.3$ as shown in figure 5, the Zener resonances are destroyed completely, provided that $X(k = 0, t)$ remains positive throughout the whole driving process. The oscillations on plateaus which include two periods of the oscillation can still be identified in earlier plateaus, but their amplitudes decrease with increasing time. For the case of strong noise ($D/\omega_B = 1.0$, $\omega_B \tau_c = 1.0$), not only the Zener oscillations but also the plateaus are destroyed almost completely as shown in figure 6.

Figure 7 shows the time dependencies of $X(k = 0, t)$ in the absence of noise and for the case of strong noise with $D/\omega_B = 1.0$, $\omega_B \tau_c = 1.0$, where the superlattice parameters are chosen to be the same as those for figure 4, but the static part of the electric field is chosen to be well away from the avoided crossings. In the absence of noise, the electrons are localized in one of the minibands. In the case of strong noise, the miniband localization is destroyed completely, provided that $X(k = 0, t)$ decays almost exponentially to zero with increasing time.

From the above results, we can see that the external noise can destroy not only the Zener resonances, but also the BOs (the plateaus are the traces of BOs in the two-band model). The strength of the noise required to destroy the Zener resonance is much less than that required to destroy the BOs. This will be manifested through comparing the strength of the noise relative to the Zener resonance frequency with that relative to the BO frequency. Since the Bloch frequency ω_B is much larger than the Zener resonance frequency Ω_Z, for a given strength of the noise D, D/Ω_Z is much larger than D/ω_B. Thus, the destruction of the Zener resonances is much easier than that of the BOs.

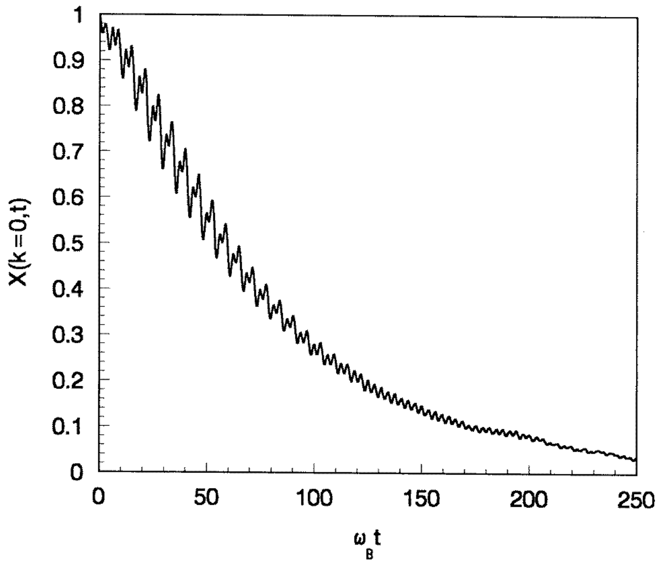

Figure 5. $X(k=0,t)$ with external noise as in figure 4, but for $D/\omega_B = 0.3$.

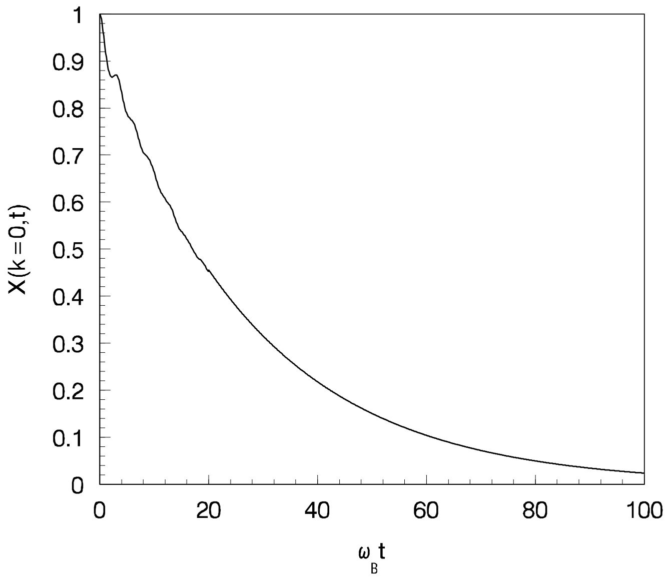

Figure 6. $X(k=0,t)$ with external noise as in figure 4, but with $D/\omega_B = 1.0$ and $\omega_B\tau_c = 1.0$.

The problem that we study in this paper is an infinite one-dimensional model. As is well known, there is some subtlety in this problem because the system is unstable due to the external electric field [35]. A rigorous mathematical theorem [36, 37] has already shown this point. However, if we only consider groups of finite numbers of bands (the N-band approximation), the system becomes stable [34]. Physically speaking, it is enough to consider finite numbers of bands for practical situations. Therefore, our results obtained in this paper, which are based

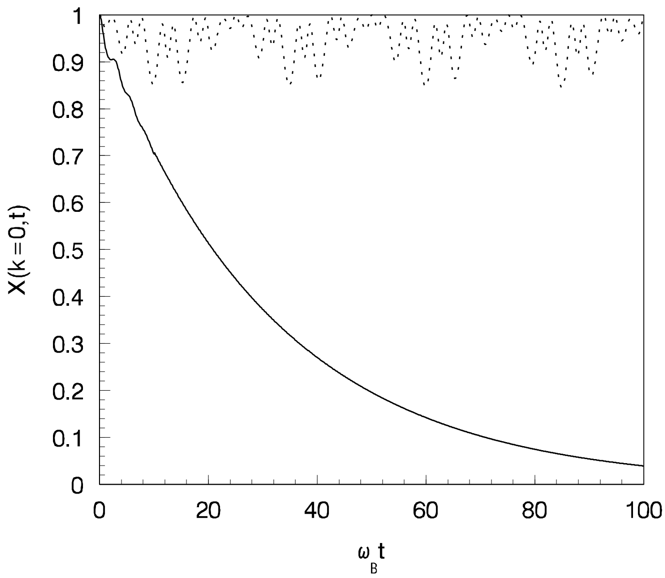

Figure 7. $X(k = 0, t)$ as a function of dimensionless time $\omega_B t$ with and without external noise. $W = 8.6$ meV, $\Delta_{ab} = 20$ meV, $R/d = 0.18$, $\omega_B = 9$ meV, $D/\omega_B = 1.0$, and $\omega_B \tau_c = 1.0$. The solid line corresponds to the case of strong noise, and the dotted line to the case without external noise.

on the standard tight-binding two-band model [23], will have potential application to quantum physics.

In summary, we have studied the problem of transport properties of two-band superlattices under the influence of an external electric field. The characteristic of our model is that we have introduced a stochastic part of the field via an OU process to describe the effect of external noise. We found that

(i) When the strength of noise is weak, the Zener tunnelling and the BOs can be displayed by the oscillations between negative and positive values; nevertheless their amplitudes decrease with increasing time.

(ii) When the strength of the noise increases to a certain value, the BOs are still displayed by the plateaus of $X(k, t)$, but the Zener resonances are suppressed heavily.

(iii) When the strength of the noise is strong, not only the Zener resonances but also the BOs will be destroyed completely.

(iv) The miniband localization can also be destroyed by the noise.

Considering these observations, in order to observe the Zener tunnelling oscillatory behaviour, one should produce a stable electric field which has the smallest possible stochastic component.

Acknowledgments

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